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# Heterogeneous R&D spillovers and sustainable growth: Limits to efficient regulation*

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**Abstract**

This paper introduces heterogeneity of cross-technologies interactions into the double-differentiated R&D-based endogenous growth model. In this model new technologies appear continuously and older are outdated generating structural change. All technologies may interact with each other through knowledge spillovers which are technology-specific and this results in innovations' heterogeneity. The conditions on the shape of these interactions for the existence of the (sustained) growth path in the decentralized economy as well as for the social planner's problem are established. Next the necessity for government interventions depending on the complexity of these interactions is studied. At last the scale and duration of interventions are demonstrated to be functions of spectral properties of the interactions operator.

**JEL classification**: C61; O32; O38; O41; H3

**Keywords**: endogenous structural change; endogenous growth; technological spillovers; R&D policy; government regulation; dynamic stability; spectral theory; optimal control

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# 1 Introduction

The process of large-scale technical change involves not only the emergence of new technologies and refinement of existing ones, but also the structural change of the economy whereas some older technologies and sectors are replaced by newer ones. This is the case with renewable energy transition being actively pursued by many European economies and the modernization of developing economies where traditional sectors are gradually replaced by the more technologically-intensive ones.

Once such a structural change is of an issue it is important not only to understand the drivers of this change, but also the impact of the changing structure of knowledge spillovers being experienced by the R&D sector. Existing growth models fall short of capturing both these issues. First, majority of the growth literature assume a rather simplistic structure of knowledge spillover. In particular, the intensity of spillovers is assumed to be uniform across technologies, even if dependent on the existing number of technologies, as in Peretto and Connolly (2007),Acemoglu et al. (2012) among others. The recent exceptions are Acemoglu and Cao (2015) and Chu et al. (2017) where firms' heterogeneity is allowed for but this is not attributed to the structure of R&D spillovers as a whole.

Second, the exit of outdated technologies is rarely accounted for and the range of technologies is either stabilizing in the long-run, resulting in the vertical innovations being the primary growth driver as in Peretto and Connolly (2007), or the range of sectors grow in an unlimited way, as in Chu et al. (2012). In recent Hamano and Zanetti (2017) endogenous process of firms' entry and exit is modelled, but continuous structural change is not the growth driver there.

In this paper, we advance and study a model of cross-technology interactions that is more general than existing models regarding possible interdependencies of technological developments and allows for almost any type of pairwise cross-technologies dependencies. At the same time to model the large-scale technological transition we make use of the endogenous growth model of structural change, Bondarev and Greiner (2017) whereas new technologies arrive continuously and older ones are scrapped due to competitive pressure and limited resources available. The paper thus combines this novel endogenous structural change framework with heterogeneity of knowledge spillovers.

The contribution of this paper is threefold. First, the general properties of such a heterogeneous spillovers are studied. It is shown under which conditions the underlying economy would possess balanced growth paths in decentralized and centralized cases. It turns out that for market economy those conditions are very restrictive, whereas for a social planner they are more relaxed.

Second we find out that the first-best solution for the economy is not always feasible even without liquidity constraints for the government. Conditions for the feasibility of the first-best structural change are established based on the spectral properties of cross-technologies interactions.

At last the characterization of the size and duration of regulation necessary to grant dynamic consistency ot such an economy in technological transition are obtained.

In the next section, the model is described. Section 3 contains main results of the paper. In particular I establish the balanced growth path existnece conditions (Subsec. 3.1), show how and why the decentralized solution and a social planner's solution diverge (Subsec. 3.2), and when the government intervention suffices to restore the balanced growth path (Subsec. 3.3). Section 4 concludes.

# 2 Model

The framework is based on the endogenous structural change model with symmetric technologies, described in Bondarev and Greiner (2017), where the reader is referred to for details. Here only essential model ingredients are described to make exposition self-contained. The main novelty of the suggested model lies in the introduction of a fairly general way of the heterogeneity of technologies through the cross-technologies interactions operator, similar to Bondarev and Krysiak (2017) where only partial equilibrium setting is considered.

## 2.1 Households

Households are modelled in a standard way. The amount of labour is constant and distributed across the range of final sectors, which are in existence:

$$L = \int\limits_{N_{min}(t)}^{N_{max}(t)} L(i,t)di,$$

$$N_{min}(t) < N_{max}(t) < N(t), \tag{1}$$

where:

- $L$ is the total labour in the economy (equal to population),
- $L(i)$ is the employment in sector $i$,
- $N(t)$ is the number of products or technologies (range) invented up to time $t$,
- $N_{max}(t)$ is the range of manufacturing sectors with positive operating profit (any new technology does not immediately yield positive productivity),
- $N_{min}(t)$ is the range of sectors, which have disappeared from the economy up to time $t$.

The objective functional of the household (lifetime utility) is

$$J^H = \int\limits_0^{\infty} \mathbf{e}^{-\rho t} U(C)dt, \tag{2}$$

with $U(C) = \ln C$ being the utility function from composite consumption $C$ consisting of the continuum of products,

$$C = \left[ \int_{N_{min}}^{N_{max}} C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

with $1 < \varepsilon < \infty$ being the elasticity of substitution between goods.

The flow budget constraint of the household is

$$\dot{a} = ra + L - \int_{N_{min}}^{N_{max}} P_i C_i di - Ta, \tag{4}$$

with $L$ the numeraire so that the wage rate is equal to one and where:

- $a$ is the value of assets being hold by the households
- $r$ is the interest rate
- $T$ is the income tax rate (time-varying or not)

We assume zero depreciation rate of capital for simplicity. Positive depreciation will not essentially change the results of the paper. We denote consumption expenditures by $E$:

$$E = \int_{N_{min}}^{N_{max}} P_i C_i di \; , \tag{5}$$

along the same range of existing sectors to condense notation.

Consumption of the individual good $i$ is given by

$$C_i = E \frac{P_i^{-\varepsilon}}{\int_{N_{min}}^{N_{max}} P_j^{1-\varepsilon} dj} \; . \tag{6}$$

The standard Euler equation implies that the optimal growth rate for expenditure is given by

$$\frac{\dot{E}}{E} = (r - T) - \rho \; , \tag{7}$$

## 2.2 Goods Producers

Goods producers employ labour and buy technology from the R&D sector. With these inputs they produce the goods which they sell to the consumer. Output of good $i$ is given by:

$$Y_i = A_i^{\alpha} L_i \; , \tag{8}$$

where $0 < \alpha < 1$ determines the productivity of the technology in production. The productivity $A_i$ is the result of vertical innovations that raise the quality of a given technology and that are generated by the R&D sector.

The profit of firm $i$ is

$$\Pi_i = P_i Y_i - L_i - \Psi \; , \tag{9}$$

where $\Psi$ is a fixed operating cost.[1]

[1] check whether we need them; seems nmax equals n without them, why then?

The only use for output of all goods $i$ is consumption, so that $C_i = Y_i$. The only product used for investments is financial capital $a$ which is excluded from this spectrum. Firm $i$, therefore, sets its price to

$$P_i = \frac{\varepsilon}{\varepsilon - 1} A_i^{-\alpha} \ . \tag{10}$$

This is the price defined only for the products in the range $N_{max} - N_{min}$. All products out of the range $N_{max} - N_{min}$ have a price of zero:

$$P_i = \begin{cases} 0,\ t < \tau_{max}(i), \tau_{max}(i) : \Pi_i = 0, \dot{\Pi}_i > 0, \\ \frac{\varepsilon}{\varepsilon-1} A_i^{-\alpha},\ \tau_{max}(i) < t \leq \tau_{min}(i), \tau_{min}(i) : \Pi_i = 0, \dot{\Pi}_i < 0, \\ 0, t > \tau_{min}(i). \end{cases} \tag{11}$$

Here and throughout the paper we use the following notation:

- $\tau_{min} = N_{min}^{-1}(i)$, time when product (technology) $i$ becomes out-dated and profit of manufacturing decreases below zero;
- $\tau_{max} = N_{max}^{-1}(i)$, time when product (technology) $i$ becomes profitable and manufacturing sector starts production of positive amounts;
- $\tau_0 = N^{-1}(i)$, time when technology $i$ is invented through horizontal innovations process.

Inserting (6) and (10) into (8) yields labour demand as,

$$L_i^D = \frac{\epsilon - 1}{\epsilon} E \frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{-\alpha(1-\epsilon)} dj} \ . \tag{12}$$

Labour employed in sector $i$ is thus a function of the relative productivity of labour in sector $i$. Repeating the arguments made with respect to the price formation, we get a piecewise-defined labour demand:

$$L^D(i) = \begin{cases} 0,\ t < \tau_{max}(i), \tau_{max}(i) : \Pi_i = 0, \dot{\Pi}_i > 0, \\ \frac{\epsilon-1}{\epsilon} E \frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N_{max}} A_j^{-\alpha(1-\epsilon)} dj},\ \tau_{max}(i) < t \leq \tau_{min}(i), \tau_{min}(i) : \Pi_i = 0, \dot{\Pi}_i < 0, \\ 0, t > \tau_{min}(i). \end{cases} \tag{13}$$

The technology is acquired by the goods producers in the form of a patent and the pricing for this patent follows Nordhaus (1967), Romer (1990) and Grimaud and Rouge (2004). The price of the patent (blueprint) equals the total value of profits which can be derived from it. The manufacturing firm can extract positive profits only for a limited period of time. Thus the patent price is defined as:

$$p_A(i) \stackrel{def}{=} \int_{\tau_{max}}^{\tau_{min}} \mathrm{e}^{-r(t-\tau_0)} \Pi_i dt. \tag{14}$$

The date at which patent $i$ starts, $\tau_{max}$, is endogenously determined by the productivity threshold necessary to gain positive profits, while the effective duration of the patent is endogenously determined from the demand for the manufactured product $i$, by the point in time, $\tau_{min}$, when the final producer can no longer earn positive profits. Thus, the duration of the patent is determined by two zero-profits conditions.

Further, the patent price is independent of time. It only depends on the ratio of productivity in sector $i$ at time points $\tau_{max}, \tau_{min}$. This observation directly follows the benchmark model.

## 2.3 R&D Sector

The R&D sector includes two dimensions: vertical and horizontal innovations. Both types of R&D use household assets as the only input. Thus, the total sum of both kinds of R&D investments at any time forms the demand for assets in the economy:

$$u(t) + \int_{N_{min}(t)}^{N(t)} g(i,t) di = a^D(t) \ , \tag{15}$$

where

- $u(t)$ are horizontal innovations investments at time $t$;
- $g(i,t)$ are vertical innovations investments at time $t$ for technology $i$ within the range of invented and not out-dated technologies, $[N_{min}(t), N(t)]$;
- $a^D(t)$ is the total demand for assets.

All new technologies may differ in their impact on the existing set of technologies. Thus the heterogeneity of technologies stems from cross-technologies interactions, which could be positive (spillovers) or negative (business-stealing effect among R&D firms).

Both types of investments are optimally set as strategies of associated firms in their optimal control problems. First the problem of R&D investments in horizontal innovations is described and then that of vertical innovations.

### 2.3.1 Horizontal innovations

The creation of new technologies (horizontal innovations) in general follows the setup of Peretto and Connolly (2007) and closely of Bondarev and Greiner (2017). We assume that new technologies appear due to knowledge creation mechanisms that are governed by private initiatives of competitive R&D firms. New technologies are created through R&D investments, $u(t)$, chosen optimally by the firms[2]:

$$\dot{N} = u(t) \ , \tag{16}$$

These are financed from the assets of the households $a(t)$ and represent a part of the total assets demand $a^D$ in (15).

The incentive for horizontal innovations is the potential profit from selling the technology to manufacturing firms, detailed further in (21). Assume that the horizontal R&D firm which invents technology $i$ later develops it through vertical innovations. The two-step sequential optimization is equivalent to the joint optimization in this setup, see Bondarev (2016) for example. Thus, the value of horizontal R&D consists solely in expected future profits from vertical innovations:

$$V_N = \max_{u(\bullet)} \int_0^\infty \mathrm{e}^{-rt} \left( \pi^R(i)|_{i=N} u(t) - \frac{1}{2} u^2(t) \right) dt. \tag{17}$$

[2]of course the horizontal dimension may include knowledge spillovers as in standard endogenous growth models. This is assumed away to streamline the exposition.

Here, the profit of developing the next technology $i = N$, $\pi^R(i)|_{i=N}$, equals the value of vertical innovations into technology $i$, which is given by:

$$\pi^R(i)|_{i=N} = p_A(N) - \frac{1}{2} \int\limits_{\tau_0(N)}^{\tau_{min}(N)} \mathrm{e}^{-r(t-\tau_0)} g^2(N,t) dt \ , \tag{18}$$

with $g(N,t)$ investments into the development of technology $N$ during the phase when technology $i$ has non-zero productivity. The fact that the value of a horizontal innovation depends only on the next technology is equivalent to the result of Chu (2011) on the presence of an *Arrow replacement effect*: each new technology is owned by a separate R&D firm.

Next observation concerns the free entry of R&D firms:

**Lemma 1.** *Under the free entry of firms into R&D sector (no strategic behavior of incumbents) the potential profit for each new technology is constant,* $\pi^R(i) = const$.

*Proof.* Free-entry implies that any firm may enter the horizontal innovations process at any stage. If some technology $i^*$ has higher potential profit, the potential entrant would wait until this technology would become available for research ($N$ approaches this $i^*$). But then all potential entrants would do the same for any technology with higher potential profit. This implies in the limit all technologies would have the same potential profit. □

This immediately implies that independently of the form of the cross-technologies interactions the horizontal technologies expansion remains linear as in the benchmark case:

**Corollary 1.** *The horizontal expansion of technologies' range has constant speed*

$$\dot{N} = \pi^R(N) = const \tag{19}$$

*Proof.* Construction of HJB (Hamilton-Jacobi-Bellman) equation for the problem (17) - (16) yields optimal investments as a function of $\pi^R(N)$ and marginal expansion value $\frac{\partial V_N}{\partial N}$. It then follows that once $\pi^R(N) = const$ via Lemma 1, the only value function which satisfies this HJB equation is a constant one, implying $\frac{\partial V_N}{\partial N} = 0$ and thus $\dot{N} = u = \pi^R(N) = const$. □

We then may define the resources available for vertical innovations in the same way as for the benchmark model:

$$\pi^R + \int\limits_{N_{min}(t)}^{N(t)} g(i,t)di = a^D(t);$$

$$G(t) \overset{def}{=} \int\limits_{N_{min}(t)}^{N(t)} g(i,t)di = a(t) - \pi^R \tag{20}$$

where $G(t)$ denotes the financial resources available for vertical innovations at time $t$.

### 2.3.2 Vertical innovations

Productivity-improving innovations (vertical innovations) lead to a rise in efficiency of technologies that have zero productivity upon their invention. This productivity can be developed through specific investments for every product.

Profits in R&D results from sales of blueprints to manufacturing firms. These sales come in the form of patents for each new technology $i$ and all of the investments into the development of each new technology (vertical innovations) are financed from this patent payment. Costs of R&D are costs of development of the productivity through technology-specific investments $g_i$. These investments are financed from household assets $a$ forming part of assets demand $a^D$ in (15).

The profit associated with the development of technology $i$ is given by:

$$\pi^R(i) = p_A(i) - \frac{1}{2} \int\limits_{\tau_0}^{\tau_{min}} \mathrm{e}^{-r(t-\tau_0)} g^2(i,t)dt, \tag{21}$$

with investments going into the increase of productivity as long as the technology is operational:

$$\forall i \in [N_{min}(t), N(t)] : \ \dot{A}^P(i,t) = g^P(i,t) - A^P(i,t) \ , \tag{22}$$

where superscript $P$ denotes individually optimal quantities (to distinguish them from socially optimal ones).

We first observe that the productivity of any technology is zero before it is invented and after it is not used in manufacturing:

**Lemma 2.** *For all non-operational technologies the individually optimal productivity investments are zero*

$$\forall i \notin [N_{min}(t), N(t)] : \; g^P(i,t) = 0 \tag{23}$$

*Proof.* As soon as $i > N(t)$ the technology is not yet invented and thus there is no associated manufacturing sector demand for it. Thus profit incentive is zero and investments are zero. Once $i < N_{min}(t)$ the manufacturing sector associated with technology $i$ does no longer generate positive profit and again investment incentives are zero. □

Apart from individual investments there exist externalities experienced by each R&D firm from all other existing R&D firms. This impact is described by the interactions operator $\Theta$ such that:

$$\dot{A}^{SP}(i,t) = \Theta A(j,t) \stackrel{def}{=} \int_{N_{min}(t)}^{N(t)} \Theta(i,j) A(j,t) dj \tag{24}$$

so any technology is subject to potential impact of those technologies present at $t$. The cross-technologies interactions are state-dependent and their intensity is given by the interactions operator $\Theta$ with $\Theta(i,j)$ measuring the impact of technology $j$ on technology $i$. This operator then is a natural generalization of the commonly employed knowledge spillover, which acts as a main growth driver in the majority of endogenous growth literature (see Peretto and Smulders (2002) for discussion of knowledge spillovers).

However the general operator like (24) is difficult to analyze, since it might be the case that already outdated technologies would experience a revival due to new spillovers from emerging technologies. We thus impose the simplifying assumption on the structure of cross-technologies interactions to make sure this is impossible in the model:

**Assumption 1.** *For any technology $i$ the range of technologies influencing it is limited:*

$$\forall i \in [0, \infty) : \; \Theta(i,j) = \begin{cases} \theta(i,j), \; \textit{if}\, N_{min}(\tau_0(i)) \leq j \leq N(\tau_{min}(i)), \\ 0, \; \textit{otherwise} \end{cases} \tag{25}$$

Here $N(\tau_{min}(i))$ denotes the newest invented technology at the time technology $i$ becomes outdated $(\tau_{min}(i))$ and $N_{min}(\tau_0(i))$ denotes the oldest technology at the time technology invented ($\tau_0(i)$). Assumption 25 thus requires that those technologies, which appear after $i$ is outdated, cannot influence it and those, which are already outdated when $i$ is just invented cannot impact it too. In this way the introduced assumption seems to be not very much binding.

In fact this is the statement that spillovers are sufficiently *local*: there is a limit on the range of technologies which is influenced by any given technology.

This assumption makes the operator $\Theta$ time-invariant: the overall impact on any technology varies over time because of state-dependency, but the operator of these impacts is fixed for any $i$. We thus may proceed as in Bondarev and Krysiak (2017) with analysis of cross-technologies interactions[3]. The total evolution of productivity for every (invented) technology $i \in [N_{min}, N]$ is thus a sum of controlled individual investments and the externalities impact:

$$\dot{A}^T(i,t) = \dot{A}^P(i,t) + \int_{N_{min}(\tau_0(i))}^{N(\tau_{min}(i))} \theta(i,j)A(j,t)dj \tag{26}$$

where we use Assumption 1 replacing $\dot{A}^{SP}(i,t)$ with a simpler bounded domain operator. Observe that once $j > N(t)$ the productivity for technology $j$ is zero by the Lemma 2, and thus in an effect the spillover operator takes into account only already invented technologies.

We next assume that in the decentralized economy individual R&D firms cannot track the individual impacts of different technologies on each other (non-atomicity assumption):

**Assumption 2.** *For any R&D firm $i$ the impact of cross-technologies interactions is a function of time only, independent of the state of any individual technology:*

$$\int_{N_{min}(\tau_0(i))}^{N(\tau_{min}(i))} \theta(i,j)A(j,t)dj \stackrel{def}{=} \Theta(i,t) : \frac{\partial \Theta(i,t)}{\partial A(j,t)} = 0 \tag{27}$$

Using this assumption we can set up the R&D firm problem as a standard optimal control problem in finite time (see Seierstad and Sydsaeter (1999) for example) and apply

[3]it is of interest to account for interactions operators with unbounded dynamic range (non-local ones), that is, relaxing Assumption 1. However this rises substantial technical difficulties and involve non-local operators theory, which is still under development, see e. g. Bernardis et al. (2016).

the Maximum Principle. As a result we obtain optimal investments and productivity evolution for each $i$ as:

$$g^P(i) = \begin{cases} \frac{\partial p_A(i)/\partial A_i}{1+r}(1 - \mathrm{e}^{(1+r)(t-\tau_{min}(i))}), & \text{if } \tau_{min}(i) \geq t \geq \tau_0(i), \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

$$\dot{A}^P(i,t) = \begin{cases} \frac{\partial p_A(i)/\partial A_i}{1+r}(1 - \mathrm{e}^{(1+r)(t-\tau_{min}(i))}) - A^P(i,t) + \Theta(i,t), & \text{if } \tau_{min}(i) \geq t \geq \tau_0(i), \\ -A^P(i,t) + \Theta(i,t), & \text{if } t > \tau_{min}(i), \\ 0, & \text{if } t < \tau_0(i) \end{cases} \tag{29}$$

with $\Theta(i,t)$ defined by (27) and $\partial p_A(i)/\partial A_i$ being the marginal return to the increase in productivity of $i$ in terms of the patent price. Observe that this last is time-invariant (since the patent price itself is time-invariant) for each $i$, but varies across $i$. So for mathematical derivations this quantity may be treated as constant in time.

Given this observation the evolution of each technology, (29) is a linear non-autonomous differential equation within operational phase $\tau_{min}(i) \leq t \leq \tau_0(i)$. Once $t > \tau_{min}(i)$ productivity investments are zero, but the spillover part may still affect the productivity evolution. By Assumption 1 the spillover affecting outdated technologies is limited: after $\tau_{min}(N(\tau_{min}(i)))$ (which is the time when the last invented during the development of $i$ technology becomes outdated) it is strictly zero and the technology decays to zero as in the baseline model. During the period $t \in [\tau_{min}(i), \tau_{min}(N(\tau_{min}(i)))]$ the manufacturing firm may still use the technology as it may remain competitive even in the absence of R&D investments. This type of producers we refer to as *imitators* as their behavior resembles one in Acemoglu and Cao (2015).

So every technology potentially exhibits up to three phases of dynamics:

1. Normal R&D development till $\tau_{min}(i)$ when productivity is supported by the associated R&D firm receiving patent payments;

2. Free-rider development may start at any time after $\tau_{min}(i)$ and continues at max till $\tau_{min}(N(\tau_{min}(i)))$ but can stop earlier;

3. Irreversible decay of technology after $\tau_{min}(N(\tau_{min}(i)))$

The second phase is novel and appears due to the presence of cross-technologies interactions $\Theta$ in the model. Depending on the structure of these interactions this phase may have different duration and the development of technology may even exceed that during the normal phase.

These firms would stay operational for some positive time as soon as the accumulated profit during this spillover phase is non-negative. However, this will increase the competition at the labor market. We thus assume throughout the main part of the paper that spillovers intensity is bounded in some precise sense:

**Definition 1.** *Technology $i$ is normal, if intensity of spillovers it experiences is not high enough for potential free-riders to make positive profit:*

$$\int_{\tau_{min}(i)}^{\tau(N(\tau_{min}(i)))} \mathrm{e}^{r(t-\tau_{min}(i))}\Pi_i dt \leq 0 \tag{30}$$

*Otherwise technology $i$ is free-riding.*

We thus assume

**Assumption 3.** *Operator $\Theta$ is such that all technologies are normal in the sense of Definition 1.*

It follows that once we adopt Assumption 3, every technology has only one operational cycle which corresponds to the time when investments are positive[4].

## 2.4 Government

As soon as there exist externalities across R&D, characterized by the operator $\Theta$, there might be a need for government interventions. To account for this opportunity we include a government into the model, thus diverging from the benchmark case.

The government is caring only for market failures corrections, thus social planners' problem is to maximize welfare coming from R&D only:

$$J^G = \max_{g(\bullet),u(\bullet)} \int_0^\infty \mathrm{e}^{-\rho t} \left\{ \int_{N_{min}}^{N} \mathrm{e}^{-\rho(t-\tau_0(i))} \left\{ b(i) - \frac{1}{2} g^2(i,t) \right\} di - \frac{1}{2} u^2(t) \right\} dt \tag{31}$$

[4]again it would be of interest to allow for free-riding technologies, but this may lead to chaotic and even non-deterministic dynamics because of piecewise-smooth system resulting from (29), see e. g. Colombo and Jeffrey (2011) for details.

with $b(i)$ being the *social value of the technology* $i$. The focus of this paper is the market inefficiency stemming from cross-technologies interactions so I make a simplifying assumption concerning the social value of technologies, letting $\forall i \in [0, \infty): \; b(i) = \pi^R(i)$. All the main results would hold for $b(i) \neq \pi^R(i)$ under suitable assumptions on this social welfare wedge (like boundedness) but will unnecessarily overcomplicate the exposition.

The objective (31) together with constraints (15), (26), (16) defines the optimization problem for the government which would yield a first-best schedule of technologies' development. Next, once this is different from the market solution, government would need subsidies (positive or negative) to different technologies. If this would be the case, we require government to run a balanced budget[5]:

$$\forall t \in [0, \infty): \; S(t) \stackrel{def}{=} \int_{N_{min}}^{N} \{s(i,t)di + s(N(t),t)\} \, dt = T(t)a(t) \tag{32}$$

where $S(t)$ is defined to be a total sum of subsidies (positive or negative), $T(t)$ is the income tax rate levied on households' assets[6]. Once $S(t) \neq 0$ the government optimization problem has to take (32) into account.

The social planner's solution differs from the decentralized one exactly by the impact of each technology on all others. Formally the social planner's problem is maximizing (31) subject to (16) and (22). This constitutes an infinite-dimensional infinite-horizon optimal control problem. As soon as $\tau_{0,min,max}(i)$ are taken as given the problem is equivalent to the one considered in Bondarev (2015). Using Maximum Principle as of Skritek et al. (2014), we may derive socially-optimal horizontal and vertical investments. The time of emergence and profitability of technologies are then obtained as inverse functions of

[5]we neglect the possibility of deficit-run budget for the sake of simplicity, but again this would not influence the main results of the paper

[6]it has been shown by Greiner and Bondarev (2015) that consumption tax actively discussed recently may cause instability of the taxed economy so we limit exposition to the income tax only

variety expansion:

$$
\begin{aligned}
&g^*(i,t) = \psi^*(i,t) = \psi^R(i,t) + \frac{1}{1+r}(1 - \mathrm{e}^{(1+r)(t-\tau^*_{min}(i))}) \int_{N_{min}}^{N_{max}} \Theta(j,i)\psi^*(j,t)dj,\\
&\dot{A}^*(i,t)) = \psi^*(i,t) - A^*(i,t) + \int_{N_{min}}^{N_{max}} \Theta(i,j)A^*(j,t)dj,\\
&u^*(t) = \lambda^*(t),\\
&\dot{N}^*(t) = \dot{N}^*_{min}(t) = \dot{N}^*_{max}(t) = \lambda^*(t),\\
&\tau^*_0(i) = (N^*(t)|_{N=i})^{-1}, \tau^*_{min}(i) = (N^*_{min}(t)|_{N=i})^{-1}, \tau^*_{max}(i) = (N^*_{max}(t)|_{N=i})^{-1} \qquad (33)
\end{aligned}
$$

In (33) we used the fact that outdating and operational phase entering of technologies are proportional to the emergence of new ones (otherwise the economy would collapse in finite time) and once $b(i) = \pi^R(i)$ the variety expansion is still linear. Then we get

**Lemma 3** (Timing lemma)**.**
*Under the assumption $b(i) = \pi^R(i)$ and free entry the timing for all new technologies coincide under social planner's and market solutions:*

$$\forall i \in [N_0, \infty): \ \tau^*_0(i) = \tau^P_0(i),\ \tau^*_{min}(i) = \tau^P_{min}(i),\ \tau^*_{max}(i) = \tau^P_{max}(i). \qquad (34)$$

*This provides consistency for government policy.*

*Proof.* Once free entry holds, we get $\pi^R(i) = C$. Then under social planner's regime we still get constant returns to every technology. This grants linear expansion rate and thus equal timing for both economies. □

This lemma is necessary to implement policies, since otherwise the mass of technologies in operational phase/in existence would not coincide and any policy instrument targeted at more than a single technology would be ill-posed.

Once Lemma 3 holds it is immediate to design the first-best subsidies schedule:

**Lemma 4.** *The first-best subsidy to R&D is given by*

$$s(i,t) := \frac{1}{1+r}(1 - \mathrm{e}^{(1+r)(t-\tau^*_{min}(i))}) \int_{N_{min}}^{N_{max}} \Theta(j,i)\psi^*(j,t)dj \qquad (35)$$

*It is feasible as long as $\Theta$ is a compact operator and (32) holds.*

*Proof.* Comparing decentralised and centralised solutions (29) and (33) we observe that investments differ solely by the term $\frac{1}{1+r}(1-\mathrm{e}^{(1+r)(t-\tau^*_{min}(i))})\int_{N_{min}}^{N_{max}}\Theta(j,i)\psi^*(j,t)dj$. It thus suffices to assign a subsidy at this level to each technology to correct the decentralised solution to the first-best one.

This subsidy is well-defined for each $i$ only if the associated integral equation (Fredholm equation) $\psi^*(i,t) = \frac{\partial p_A(i)/\partial A_i}{1+r}(1-\mathrm{e}^{(1+r)(t-\tau_{min}(i))})+\frac{1}{1+r}(1-\mathrm{e}^{(1+r)(t-\tau^*_{min}(i))})\int_{N_{min}}^{N_{max}}\Theta(j,i)\psi^*(j,t)dj$ has a solution. This is the case as long as $\Theta$ is a compact operator and $\psi^*(i,t)$ is defined for each $i$.

At last, the budget constraint has to hold for the subsidy to be feasible, i. e. there are sufficient funds to cover all additional expenditures. □

Lemma 4 states that first-best subsidy schedule exists, but the requirement of compactness is rather strong, since the operator changes its value every time new technology arrives. In the absence of structural change this is almost always the case, since $\Theta$ becomes time-invariant. However under structural change as it is understood in this paper, first.best subsidies are not always feasible, as the following analysis demonstrates.

To complete the description of the model we list market clearing conditions, which are exactly the same as in the benchmark model.

## 2.5 Markets clearing

First, observe that $\dot{E} = 0$, since prices are moving in the opposite direction of productivities, total labor force is constant and labor income is a numeraire. Then the expenditures are a cosntant fraction of the labor income:

$$E = \frac{\epsilon}{\epsilon - 1}L \tag{36}$$

implying more or less that the labor income is consumed depending on $\epsilon$ value (for $\epsilon > 2$ some capital income is consumed, for $2 > \epsilon > 1$ fraction of labor income is saved).

Next, using $\dot{E} = 0$ and the Euler equation, we can derive the interest rate in equilibrium:

$$\frac{\dot{E}}{E} = r - T - \rho = 0 \rightarrow r = T + \rho. \tag{37}$$

For the real interest rate to be constant, taxes should be constant in time also, implying the requirement of government dynamic consistency.

Labour market clearing condition is given if the following holds:

$$\int\limits_{N_{min}}^{N} L^D(i,t)di = L = L\int\limits_{N_{min}}^{N} \frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N} A_j^{-\alpha(1-\epsilon)}dj}di,$$

$$\int\limits_{N_{min}}^{N} \frac{A_i^{-\alpha(1-\epsilon)}}{\int\limits_{N_{min}}^{N} A_j^{-\alpha(1-\epsilon)}dj}di = \frac{\int\limits_{N_{min}}^{N} A_i^{-\alpha(1-\epsilon)}di}{\int\limits_{N_{min}}^{N} A_j^{-\alpha(1-\epsilon)}dj} = 1. \tag{38}$$

But this last condition is automatically satisfied, hence the labour market is cleared.

At last assets are growing in the economy once the initial endowment is sufficiently high and taxes are low enough:

$$\dot{a} = (r-T)a - \frac{1}{\epsilon - 1}L, \tag{39}$$

which can be solved to obtain the assets as a function of time,

$$a(t) = \mathrm{e}^{(r-T)t}\left(a_0 - \frac{1}{(\epsilon-1)(r-T)}L\right) + \frac{1}{(r-T)(\epsilon-1)}L. \tag{40}$$

Assets accumulation is positive as long as the initial assets of households are sufficiently large:

$$a_0 > \frac{1}{\epsilon-1}\frac{1}{r}L. \tag{41}$$

As long as (41) holds, assets increase exponentially. Since horizontal investments are constant implying at most linear growth of assets demand from the horizontal R&D we have under assumption of horizontal R&D having the priority[7]

**Lemma 5.** *As long as* (41) *holds, $T < r$, then*

$$\exists! \tau_{suff} :\ \forall t > \tau_{suff} :\ G(t) > 0, \tag{42}$$

*Proof.* Follows from the fact that $a(t)$ grows exponentially and $N(t)$ linearly. There exists at most one intersection point of two monotonically growing functions of such types. Denote it $\tau_{suff}$ and the result follows. □

[7] if on the contrary, vertical R&D are prioritized, there is no structural change and the economy follows the setup of Peretto and Connolly (2007) with productivity growth only.

# 3 Results

In this section we analyze the model described above. First the long-run behavior of both decentralized and socially optimal economies are described and then we discuss the necessity, scale and duration of interventions.

## 3.1 The BGP existence conditions

We first briefly state results similar for benchmark and heterogeneous cases without proofs and then add some new insights appearing due to the spillover operator $\Theta$.

**Proposition 1.** *The productivity of the oldest operational sector, $A_{N_{min}}$, is equal to the productivity of the newest operational sector, $A_{N_{max}}$, at the time when the first is leaving the economy and the latter is entering its operational phase:*

$$A_{N_{min}} = \left( (\Psi/L)(\epsilon - 1) \int\limits_{N_{min}}^{N_{max}} A_j^{\alpha(\epsilon-1)} dj \right)^{1/\alpha(\epsilon-1)} = A_{N_{max}}. \tag{43}$$

*At the same time, the productivity of each sector grows within its operational phase,*

$$A_i(\tau_{min}(i)) > A_i(\tau_{max}(i)). \tag{44}$$

*Proof.* This follows from the zero profit condition defining the operational phase for any $i$. $\square$

Since profit is zero on both ends of the operational phase and it is nonnegative in between it follows that

**Lemma 6.** *For any technology $i$ exists at least one point $\bar{\tau}(i)$ such that the profit of the manufacturing sector is at maximum and it holds*

$$\dot{\Pi}(i) = 0 \Leftrightarrow \left( \frac{\dot{A}(i,t)}{A(i,t)} - \int\limits_{N_{min}}^{N_{max}} \frac{\dot{A}(j,t)}{A(j,t)} dj \right) = \frac{\Psi}{\alpha L}(\dot{N}_{max} - \dot{N}_{min}). \tag{45}$$

*Proof.* By mean value theorem, see any analysis textbook, as Stromberg (1981). $\square$

By Lemma 6 there are exactly two options for (45) to hold: either both sides simultaneously equal zero (which is the case for the baseline symmetric model) or generically,

$$\forall i \in [0, \infty] : \exists \bar{\tau}(i) : \frac{g_{A(i)}(\bar{\tau}(i)) - \bar{g}_A(\bar{\tau}(i))}{\dot{\mathcal{O}}(\bar{\tau}(i))} = \frac{\Psi}{\alpha L} \tag{46}$$

where we denote $g_{A(i)} \overset{def}{=} \frac{\dot{A}(i,t)}{A(i,t)}$ the growth rate of productivity for technology $i$, $\bar{g}_A \overset{def}{=} \int\limits_{N_{min}}^{N_{max}} \frac{\dot{A}(j,t)}{A(j,t)} dj$ the average growth rate of productivities and $\dot{\mathcal{O}} \overset{def}{=} \dot{N}_{max} - \dot{N}_{min}$ the shift describing the change in the size of the economy.

We call the technology, which reaches at $t$ maximum profit the *leading* technology. We infer the following observation from (46):

**Corollary 2.** *As soon as the growth rate of the leading technology is below average, the economy shrinks in size and vice versa.*

*Proof.* Indeed, since $\frac{\Psi}{\alpha L}$ is always positive, it follows that once the numerator of the righthandside in (46) is negative, the denominator is negative too, implying $\dot{N}_{max}(\bar{\tau}(i)) < \dot{N}_{min}(\bar{\tau}(i))$. This means the economy shrinks in size. With above average growth rate the opposite holds. □

It then follows that the output growth rate in the heterogeneous economy includes additional element defined by the Corollary 2 and can be non-monotonic and vary in sign:

**Corollary 3.** *The output growth may be positive or negative in the economy with heterogeneous spillovers and is given by:*

$$g_Y \overset{def}{=} \frac{\dot{Y}}{Y} = \alpha \bar{g}_A (N_{max} - N_{min}) + \frac{\Psi}{\alpha L}(\dot{N}_{max} - \dot{N}_{min}) \gtreqless 0 \tag{47}$$

*Proof.* Follows the same lines as in the baseline model for positive growth except that condition $\dot{N}_{max} = \dot{N}_{min}$ does not always hold. It then follows that once economy shrinks, growth may be negative. □

In particular, if the growth rate is positive together with the core expansion $\dot{\mathcal{O}} > 0$, such type of growth is not sustainable: as long as the expansion happens long enough as for $t^E : N_{max}(t^E) - N_{min}(t^E) \to \infty$ to realize, the growth becomes undefined, since the economy would consist of the infinitely many infinitely small sectors with infinitely low productivity.

On the other hand, if the second term is negative, implying core of the economy shrinks, this type of growth, even with positive growth rates, is also unsustainable in the long-run: as soon as for $t^S : \ N_{max}(t^S) - N_{min}(t^S) = 0$ new sectors become outdated at the very same moment they become operational. Then the long-term growth rate is zero, since $N_{max} \geq N_{min}$ cannot be violated by definition of these quantities.

We thus specify what we mean by the balanced growth path (BGP) making use of this discussion:

**Definition 2.** *The BGP of the economy described in Section 2 is the path along which two conditions hold:*

1. *The output growth rate is positive and constant*

$$\forall t \in [t^0, \infty): \ g_Y \geq 0, \dot{g}_Y = 0 \tag{48}$$

2. *The economy's size stays positive and finite:*

$$\forall t \in [t^0, \infty): \ 0 < \mathcal{O}(t) < \infty \tag{49}$$

The first condition (48) is the standard one, implying the economy grows in the long run. It does not require all the variables to grow at the same rate, but only the output, since assets and productivity would then automatically follow some balanced growth pattern (see e. g. Barbier (1999) for close definition of BGP).

The second condition is novel and reflects the importance of structural change: as soon as the number of technologies becoming operational exceeds the number of becoming outdated on a regular basis, the size of $N_{max} - N_{min} = \mathcal{O}$ grows without bounds and the growth becomes unsustainable. One may speak of the *over-diversification* of the economy: there are too many different technologies/sectors so the limited labour force is not enough to keep them running. At the same time, if the opposite happens, $N_{max} = N_{min}$, there is only one technology present in manufacturing at any time and it varies continuously, making it impossible to invest into the rise of productivity. In this case although vertical R&D takes place rising the productivity of new technologies, these technologies cannot stay operational long enough to provide positive profits for manufacturers and thus a stimulus for further inventions. This is the situation of *overburning*: the structural change intensifies so much, that older technologies are scrapped faster, then the economy may compensate them with newer ones.

We thus see than the only usual BGP in this economy is exactly the one described by the benchmark model with constant size of the core, $\dot{\mathcal{O}} = 0$. Still, this may happen only in a very special case as the following Proposition 2 shows.

It uses the operator spectral theory to some extent, so for exposition to be self-contained I list here some additional definitions.

**Definition 3.** *The spectrum of* $\Theta$*, denoted* $\sigma(\Theta)$ *is the set of* $\lambda$ *such that for any* $x$ $\sigma(\Theta) = \{\lambda : \lambda x = \Theta x\}$*.*

*The spectral radius of operator* $\Theta$*, denoted* $\rho(\Theta)$ *is the maximal absolute size of its spectrum*

$$\rho(\Theta) \overset{def}{=} \max\{|\lambda|\}. \tag{50}$$

In particular for spillover operators defined over the Hilbert space we have $\sigma(\Theta) = \sigma_p \cup \sigma_c \cup \sigma_r$ where subscripts $p, c, r$ denote pointwise, continuous and residual spectral components respectively and $\rho(\Theta) = ||\Theta||_{op}$, spectral radius equals the operator norm of $\Theta$ which is its maximal value (see e. g. Kolmogorov and Fomin (1999)).

**Definition 4.** *The operator* $\Theta$ *is scalar, if* $\forall i \neq j : \ \theta(i,j) = 0$ *and* $\forall i : \ \theta(i,i) = \theta > 0$*, i. e. it is a scalar multiple of the identity operator.*

This definition simply resembles that of a scalar matrix but for possibly infinite-dimensional setting.

**Definition 5.** *The operator* $\Theta$ *is of scalar type, if it admits the resolution of identity similar to the multiplication operator.*

We thus call $\Theta$ scalar-type if it resembles either the infinite-dimensional diagonal matrix, or by proper choice of the eigenbasis may be transformed into such a diagonal matrix (multiplication operator).

**Definition 6.** *The operator $\Theta$ is nilpotent if $\exists n \in \mathbb{N} : \Theta^n = 0$. It is topological nilpotent if $\sigma(\Theta) = 0$.*

Observe that these two notions coincide only for finite-dimensional operators.

At last we follow Dunford (1954) and define

**Definition 7.** *The operator $\Theta$ is spectral, if it admits the canonical decomposition into the scalar-type and nilpotent parts.*

In what follows we assume:

**Assumption 4.** *The spillover operator $\Theta$ is a spectral one.*

We are now ready to characterize the BGP existence of the decentralized economy.

**Proposition 2** (On BGP existence for decentralised economy with spillovers)**.**
*Assume $\Theta \neq 0$ and all spillovers are non-negative, $\forall \{i, j\} : \theta_{i,j} \geq 0$. Then:*

1. *As long as $\Theta$ is a scalar operator, the decentralized economy always possesses a BGP with constant growth rates. It is defined by the spillover size as*

$$\bar{g}_A^{\theta} = \begin{cases} \theta - 1, & \text{if } \theta > 1; \\ 1, & \text{if } \theta = 1; \\ r, & \text{if } \theta < 1. \end{cases} \tag{51}$$

2. *As long as $\Theta$ is a scalar-type operator, the decentralised economy possesses a BGP with constant growth rate independent of the spillover size $\bar{g}_A^0 = r$ if $\rho(\Theta) \leq 1$ and no BGP in the sense of Def. 2 otherwise;*

3. *If $\Theta$ is not a scalar-type operator, there is no BGP type Def. 2 in the decentralized economy with heterogeneous spillovers.*

*Proof.* see Appendix A. $\square$

Observe that Proposition 2 implies that even diagonal (multiplication) operators may cause unbalanced growth: for this to be the case it suffices for at least some technologies to experience sufficiently strong spillovers ($\theta(i) > 1$). In this case the growth rate of every technology may converge to a constant, but the average rate will vary over time, causing fluctuations in the output growth rate.

Next, we observe that for the social planner problem the BGP can be achieved in a wider variety of situations than for the decentralized economy:

**Proposition 3.** *The socially optimal R&D system* (33) *admits the balanced growth path in a sense of Definition 2 as soon as either:*

1. *the operator $\Theta$ is of the scalar-type*
2. *Operator $\Theta$ is compact.*

*Otherwise no socially optimal BGP exists.*

*Proof.* see Appendix B ☐

We thus observe that the first best solution admits the BGP as a planned one in a wider variety of cases than the decentralized economy. So the next natural question is how much inefficiency is implied by the divergence of the market economy from the centralized one.

At last define a weaker notion of sustained growth path:

**Definition 8.** *The economy follows a sustained growth path if* (49) *holds, but not necessarily* (48)*.*

It is immediate to observe that once the economy is on the sustainable growth path (SGP), it might experience prolonged periods of negative and positive growth, but the requirement (49) implies that long run growth stays positive and finite *on average*, so this economy may sustain growth for infinite time.

## 3.2 Dynamic inefficiency

In this part of the paper the potential inefficiencies of both decentralized and centralized economy are studied.

### 3.2.1 Market inefficiency

Proposition 2 states that the decentralized economy admits the constant sustained growth path only as long as spillovers operator $\Theta$ is scalar or of the scalar-type and has limited size (spectral radius is small enough). Now observe that typically dynamic nature of technologies' spectrum implies that the spillovers' structure changes continuously. We thus need some additional machinery to tackle with this issue.

Consider $\Theta|_{t^0}$: restriction of the spillover operator to the (fixed) time instant $t^0$. This restriction is a standard operator over the space of technologies, independent of time and thus it makes sense to define the compactness[8] of this restriction:

**Definition 9.** *Operator $\Theta$ is said to be $t^0$-compact if its restriction to $t^0$ is compact in all existing at $t^0$ technologies $\mathcal{J}|_{t^0} := \{i|N_{min}(t^0) \leq i \leq N(t^0)\}$.*

*Operator $\Theta$ is said to be $t^0$-scalar(-type) if its restriction to $t^0$ is scalar(-type) in all existing at $t^0$ technologies $\mathcal{J}|_{t^0}$.*

Following Proposition 2 denote by $\mathcal{F} \subseteq \mathbb{R}_+$ those time instances $t$ when operator $\Theta$ is $t$-scalar-type with small spectral radius or just scalar:

$$\mathcal{F} \stackrel{def}{=} \{t: \ \rho(\Theta(i,j)|_t) \leq 1\} \tag{52}$$

and by $\mathcal{G} \subseteq \mathbb{R}_+$ those time instances when the operator does not admit $t$-scalar form. Then it follows that at any time $t$ the spillover may take one of two forms: either it admits consistent decentralised solution or not.

Denote further by $\mathbf{S}$ the form of the spillover operator at $t$ which admits dynamically consistent market solution (e. g. scalar or scalar-type with spectral radius smaller then 1):

$$\mathbf{S} \stackrel{def}{=} \Theta|_{t\in\mathcal{F}} \tag{53}$$

**Proposition 4** (Market failure).
*For any $t \in [0, \infty)$ :*

[8] The linear operator is compact if it maps bounded subsets of the domain into relatively compact subsets of its range, see e. g. Kolmogorov and Fomin (1999).

1. *As long as* $\forall t \in \mathcal{F} \subseteq [0, \infty) : \ \Theta|_{t \in \mathcal{F}} = \mathbf{S}$ *market solution grants sustained positive long-run growth rates to the economy and no government intervention is needed.*

2. *As soon as* $\exists t_S \in \mathcal{G} \subseteq [0, \infty) : \ \Theta|_{t \geq t_S} \neq \mathbf{S}$*, market solution leads to the collapse of the economy in finite time and government intervention is necessary for some positive duration starting from* $t_S$ *onwards.*

*Proof.* See Appendix C □

*Comment:* The last property is widely known as the knife-edge property of endogenous growth models: once finely tuned conditions are violated, the economy cannot return to the balanced growth path. For discussion of these see e. g. Peretto and Valente (2015). The main novelty of the Proposition 4 lies in its dynamic nature: market economy can be efficient and sustainable for some time, but only until essentially cross-sectoral spillovers would appear in the economy. Once this is the case, such cross-sectoral interactions have to be dealt with. Moreover even when the technology causing initial spillover would be scrapped, it is not the case that the distortion caused by it will immediately stop, since there could be cascading persistent effects (for details see Subsec. 3.3).

From the other hand, Proposition 4 establishes the conditions for market interventions. These need not to be constant or even continuous. Rather the government should additionally intervene in a timely manner only at those time instances $t \in \mathcal{G}$ when homogeneity condition for technologies is violated. Such a result is possible only for a dynamic range of technologies. Indeed, once we set $\dot{N} = 0$ the interactions operator $\Theta$ is fixed and its current structure fully defines whether there is a need in the regulation or not. This regulation is then permanently in place (although the scale of intervention may decrease over time) and its efficiency fully resembles that of Bondarev and Krysiak (2017) where the cross-technologies interactions are time-invariant.

### 3.2.2 Government inefficiency

We next tackle the question under which conditions the government regulation may help the economy to achieve the sustained growth path. Observe that the Definition 8 does not imply the uniqueness of the SGP. On the contrary there might exist a lot of evolution paths of the economy which make the $\mathcal{O}$ operator positive and finite. I refer to the SGP as the *optimal* if it yields maximal social welfare among possible SGPs. With this in mind let us consider the dynamic efficiency of the government in achieving the optimal SGP.

**Proposition 5** (Government efficiency)**.**
*For any* $t \in [0, \infty)$ :

1. *As long as* $t \in \mathcal{G}$*, but* $\Theta_{t\in\mathcal{G}}$ *is compact, government subsidies may implement the first-best solution and sustained BGP is achieved;*

2. *As soon as* $\Theta_{t\in\mathcal{G}}$ *is not compact, but its residual spectrum is null, the government policy may help the economy to approach the optimal SGP with approximation error increasing in the size of continuous spectrum of* $\Theta_{t\in\mathcal{G}}$*;*

3. *As soon as* $\Theta_{t\in\mathcal{G}}$ *is not compact and its residual spectrum is non-empty, only the economy-wide average subsidy is welfare improving, but the economy will not converge to the optimal SGP.*

*Proof.* see Appendix D □

The Proposition 5 illustrates the fact that government has limited influence on the economy: at some times it can improve upon the market failure and return the economy on the BGP, but at other times it could be the case that *any* government intervention cannot help to stabilize the economy and economy-wide crisis follows. Apparently this would be the case when some fundamentally new technology appears (like those studied in the literature on general purpose technologies (GPT), see Bresnahan (2010) for an overview) which has impact on a broad range of dispersed sectors. This would be the case 3. of the Proposition 5. If this new technology's impact is limited and affects some isolated group of industries, the case 2. realizes and the government may at least smooth away part of this influence. In normal situation case 1. realizes, when new technologies influence existing structure of the economy in a limited way[9].

[9]compactness of the operator means it maps bounded sequences to bounded sequences, thus spillovers are smoothly distributed and bounded

In particular, both Propositions 4 and 5 pave the way to obtain non-monotonic growth rates in the otherwise fully analytic endogenous growth model. It thus may be applied to observed stylized facts concerning growth: it can be non-monotonic, growth rates may diverge across countries, government interventions are necessary but not sufficient to smooth away all fluctuations along the growth path.

## 3.3 Size and duration of regulation

We next ask the question on the duration of government intervention for a particular externality caused by the new technology. We limit attention to spectral operators only (see Def. 7), which is the fairly general class of operators for which the spectral theory is well established. Spectral operator is by definition linear and bounded (hence continuous) and admits the resolution of identity (see e. g. Dunford (1954)).

Any compact operator is spectral but not vice versa. The scalar-type operator is the immediate infinite-dimensional extension of what is called semi-simple operator, that is, the one without defective eigenvalues. The nilpotent part thus would contain all of potential complexities of the operator.

To this end we first limit attention to the compact case, where we understand compactness in the sense of Definition 9.

### 3.3.1 $t$-compact case

So assume for now economy is evolving in such a way that $\Theta$ is compact w. r. t. $\mathcal{J}|_t$ for all time $t \in [0, t_0]$. Compact operators possess point-wise non-zero spectrum and zero as a continuous spectrum, see Kolmogorov and Fomin (1999) for details. Thus this situation falls into case 1 of Proposition 5 and we may apply the property rights reform as defined below.

**Definition 10.** *The property rights reform in economy is given by canonical form of the operator $\Theta$. In particular, it assigns to each technology $i$ all of its externalities according to the spectrum of $\Theta$*

In particular this is the restatement of the result that operator $\Theta$ has resolution of identity and can be 'diagonalized' in a sense that it is unitary similar to the multiplication operator. The multiplication operator is an infinite-dimensional equivalent of a diagonal matrix. Thus this reform just reassigns shares of different technologies in such a way that newly defined entities do not have cross-technologies interactions, i. e. the spillover operator is 'diagonal'.

This property rights reform obviously makes sense only for the operator with non-zero pointwise spectrum, since otherwise it is not clear where to attribute some of externalities. Once we consider compact operators, they all have only point-wise spectrum (except zero) and thus we get full correspondence with the finite-dimensional Jordan-Chevalley decomposition (JCD, see Helgason (2001) for example):

**Lemma 7** (Infinite-dimensional JCD)**.**
*Any compact spectral operator over Hilbert space admits the canonical decomposition into the semi-simple part and the nilpotent part.*

*Proof.* Follows from definitions of the semi-simple, spectral and nilpotent operators and from the canonical decomposition of the spectral operator in Dunford (1954). □

Thus once we consider the case of compact operator government intervention always have only two components:

1. Rearranging property rights via the spectral decomposition of $\Theta$
2. Subsidies/taxes for technologies with $Re(\lambda_i) > 1$
3. Subsidies/taxes for the technologies which have spillovers entering the nilpotent part of $\Theta$

The first part is always possible once operator is compact and Lemma 7 holds. Indeed, for any semi-simple operator in finite dimensions the Jordan canonical form (JCF, see Weintraub (2008) for example) is diagonal. Then for compact spectral infinite-dimensional case we get equivalent as the resolution of identity with pointwise eigenvalues as a diagonal infinite-dimensional matrix. Moreover, the scalar-type part then exactly corresponds to the part of cross-technologies interactions which does not require any additional intervention: the reformed operator is already a scalar one. So the only instability source left is the possibly high impact of knowledge spillover, case 2 and the complexity impacts, reflected by the nilpotent part.

Now the duration of these two types of intervention is different: the knowledge spillover $\theta(i,i)$ has to be taken care of only within the operational phase of technology $i$. Indeed, this is the spillover affecting only this given technology (after redefinition of property rights) and thus once it becomes non-profitable there is no need in further regulation.

The nilpotent part, however, has to be regulated for a longer time: even once technology becomes outdated, its impact on other technologies persists through cascading impacts. This cascades are long-term persistent effects which slowly deteriorate over time. The duration of these post-effects is proportional to the number of technologies being affected (size of affected cluster) and fully vanishes only once all the technologies in this affected cluster are outdated. Denote, following Bondarev and Krysiak (2017), the size of the nilpotent part by the number of eigenvalues entering it:

**Definition 11.** *The t-complexity of the (t-compact) operator $\Theta$ is the number of eigenvalues with different algebraic and geometric multiplicities:*

$$\chi(\Theta|_t) = \sum_i^K (\mu^a(\lambda_i - \mu^g(\lambda_i))) \tag{54}$$

Observe that this definition makes sense only for the operator with point-wise spectrum (except zero), since for continuous and residual spectra notions of algebraic and geometric multiplicities are not well-defined. Still as long as we are in the compact case the notion of complexity is useful, as the following demonstrates

**Proposition 6.** *Assume $\Theta|_t$ is compact. Then:*

1. *As long as $\Theta|_t$ is of scalar-type and $\rho(\Theta|_t) \leq 1$ the government intervention for each $t$ consists solely of rearranging property rights via canonical decomposition of $\Theta$;*

2. *As soon as $\Theta|_t$ is of scalar-type but $\rho(\Theta|_t) > 1$ government regulation includes additionally subsidies $\forall i_1 : \lambda(i_1) > 1$ size the impact for the duration $\tau_{min}(i_1) - \tau_0(i_1)$;*

3. *As soon as* $\Theta|_t$ *contains non-zero nilpotent part and* $\infty > \chi(\Theta|_t) > 0$*, exactly* $\chi(\Theta|_t)$ *additional cascading subsidies are necessary with the duration for each cluster being* $\forall k \in \chi(\Theta|_t):\ \max_k \tau_{min}(i_k) - \max_k \tau_0(i_k)$

4. *As soon as* $\chi(\Theta|_t) \to \infty$ *the regulation is permanent and continuous as long as complexity stays infinite.*

*Proof.* see Appendix E. □

So we see that even in the best possible world of compact cross-technologies interactions there are cases when the first-best outcome may be achieved only with the help of continuous regulation. Still there are multiple instances where the time-limited and technology-specific intervention is sufficient. Observe that not only the size (intensity) of technological spillovers, but the structure (through the complexity measure) has crucial importance for the size and duration of these interventions.

However the interactions operator needs not to be compact. For example, the emergence of the drastic innovation (or GPT) would violate compactness assumption. We thus study what can be done in the case of a non-compact operator $\Theta$ next.

### 3.3.2 Noncompact interactions

We now specify within the set $\mathcal{G}$ those time instances when $\Theta$ is $t$-compact and when it is not. Denote

$$\begin{aligned}
&\mathcal{G}^1 := \{t \in \mathcal{G}\} : \sigma(\Theta|_t) = \sigma_p(\Theta|_t),\\
&\mathcal{G}^2 := \{t \in \mathcal{G}\} : \sigma(\Theta|_t) = \sigma_p(\Theta|_t) \cup \sigma_c(\Theta|_t),\\
&\mathcal{G}^3 := \{t \in \mathcal{G}\} : \sigma_r(\Theta|_t) \neq \emptyset,\\
&\mathcal{G} = \mathcal{G}^1 \cup \mathcal{G}^2 \cup \mathcal{G}^3,\ \mathcal{T} := [0, \infty) = \mathcal{G} \cup \mathcal{F}
\end{aligned} \tag{55}$$

where $\sigma_p(\Theta|_t), \sigma_c(\Theta|_t), \sigma_r(\Theta|_t)$ denote point-wise, continuous and residual components of the spectrum $\sigma(\Theta|_t)$ respectively.

Once $\Theta|_t$ is compact we apply Proposition 6 from above. Once it is not compact but self-adjoint, it may possess continuous, but not the residual spectrum. In other words the operator $\Theta$ at $t \in \mathcal{G}^2$ remains the spectral operator, but is no longer compact. Thus it admits the canonical decomposition into scalar-type and nilpotent parts, but not the JCD-type decomposition, since some part of the spectrum is continuous.

In this case the new technology appearing at some $t^b \in \mathcal{G}^2$ has substantial impact on an open set of pre-existing sectors, so that the boundaries of this impact cannot be determined precisely. The government regulation would include additional corrective subsidies for a group of affected technologies up to a point when the technology generating this spillover will become outdated (and thus $t$-compactness is restored). This additional subsidy however cannot be finely tuned as to grant the first-best allocation, since it is not clear to what extent the technologies in the affected group experience the externality. Thus this additional subsidy is group-specific but uniform within the affected group. This inefficiency comes into being because for the case of continuous spectrum the complexity $\chi(\Theta|_t)$ is not well-defined: one could count the number of affected clusters, but not the number of affected technologies within each cluster. The canonical decomposition yields the sum of multiplication-similar operator (which is then subject to property rights reform) and the nilpotent operator which contains all essentially complex interactions, but they are not isolated as in the compact case.

At last, once at some $t^n \in \mathcal{G}^3$ the new technology appears which is GPT-like and affects the whole structure of the economy, the interactions operator exhibits residual spectral component (i. e. is not self-adjoint and not compact). In this case there is no government policy in the class of subsidies which would allow the economy to approach the first-best BGP (since this does not exist at all).

# 4 Conclusion

In this paper the novel endogenous growth framework with dynamic structural change is used to study the role of cross-technologies interactions. These interactions are represented as a general infinite-dimensional matrix of pairwise technology interaction intensities. The overall knowledge spillover experienced by a given technology is then a result of the summing up individual impacts of all existing technologies weighted by intensity of the influence and the level of development of those technologies themselves. Such a shape of the spillover is fairly flexible and can capture the standard type of knowledge spillovers studied previously in the literature (like Peretto and Smulders (2002)) but also new types of spillovers. For example it allows for one-way (asymmetric) spillovers and heterogeneity of interactions.

It turns out that the decentralized economy would possess a balanced growth path exhibiting constant growth rates only for the very special type of knowledge spillover, which is by no coincidence the only one previously studied. These are scalar interactions describing equal spillover intensity for all the technologies dependent only of the technology itself. In all other cases the market cannot provide the dynamically consistent way of technological transition without government interventions. This result is independent of the specific model at hand, since unequal spillovers lead to competitive advantage of some of the technologies and the capital mobility ensures these are the only surviving technologies in the long run. On the other hand the first-best solution is dynamically consistent in a wider variety of cases. Namely it suffices for cross-technologies interactions to form a compact operator, i. e. all interactions are well defined on closed sets and are bounded in size. If this is the case the socially optimal solution may be achieved through a set of taxes/subsidies on the R&D eliminating those disbalancing competitive advantages.

In particular, two different regulation tools are suggested: the redefinition of property rights such that every technology becomes separated and independent of all the others and additional taxation of those sectors which grow faster then the average growth rate of the economy.

The second contribution of the paper is the general characterization of those cases, when not only the market but the social planner's solution cannot grant the dynamical efficiency to the economy. These are cases when the spillover operator is more complex and includes spillovers affecting a significant range of technologies and those for which the source cannot be identified precisely. In the latter case the balanced growth may be at least approximated whereas in the former case there is no way to achieve the first-best growth rates.

At last the characterization of the size and duration of interventions has been carried out. The time-varying economic policy impulses which depend on the spectral structure of the spillover operator in a general way are introduced. The periods of qualitatively different regulation are defined solely by the spectral structure of the spillover operator.

Despite their abstract nature, the results of this paper have immediate policy implications. First, it is crucial to take into account not only the intensity, but also the scope and structure of knowledge spillovers and technologies' interactions when designing the R&D policy.

Second, if the economy is undergoing structural change such that older technologies and sectors disappear and newer ones emerge (as is the case with large-scale clean energy transition) the decentralized economy is unlikely to achieve the balanced growth without government interventions. However some sustained growth may be achieved at least temporarily, but the balanced growth cannot be restored automatically due to distortions being brought into the system by the advance of newer technologies.

Third, the efficiency of government regulation of a large scale structural change depends crucially on the scale of such changes and the scope of affected sectors. If only isolated sectors/technologies are affected by new technologies, the conventional subsidizing policy would be efficient in restoring the equilibrium path of the economy. If significant clusters of sectors are under impact of emerging technologies, the government may be able to achieve the sustainable, but not the balanced growth and at last if the entire economy is affected by some general purpose technology, the optimal path cannot be sustained and (temporary) crisis is unavoidable. It thus seems that the large scale technical change comes at the cost of temporary slowdown in economic performance.

There are still many directions in the suggested framework open for future research. two of them are particularly intriguing. First, the impact of free-riding technologies may turn the dynamics to be even more complicated and become unpredictable, justifying the robust policy tools choice on the macroeconomic level. Second, the distribution of some technologies spillovers may be unbounded (consider fire or electricity as examples) making some spillovers non-local. Both these directions require far more complicated analytical tools to study and are left for future research.

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# A Proof of Proposition 2

*Proof.* As long as $\Theta$ is scalar in the sense of Definition 4 and its spectral radius is bounded by 1, it means it represents the scalar multiple $\theta \leq 1$ and the spillover qualitatively does not change the dynamics for the symmetric model. To see that just consider the symmetric case where $g^P(i,t) = \frac{G(t)}{N-N_{min}}$ following the baseline model and $\Theta(i,t) = \theta A(i,t)$. The long-run growth rate $\bar{g}_A$ is then independent of $\theta$ and equals $r$ by direct computations. This implies BGP exists as defined by Definition 2.

Once $\rho(\Theta) > 1$, every technology has increasing growth due to spillover. However the growth rate converges to the same value for any technology because of the turnpike property of the governing dynamical system in the same way as in the benchmark case (see e. g. Yano (1984) for details). Direct computation shows that $\bar{g}_A = \theta - 1$ in this case.

Once $\Theta$ is not a scalar, it has some heterogeneity and under assumption $\theta(i, j) > 0$ some technologies have competitive advantage and grow faster than others. Then either the economy collapses into one-sector either it experiences explosive growth as follows from Corollary 2. In both cases no BGP exists and sustained growth requires government interventions. □

# B Proof of Proposition 3

*Proof.* As soon as cross-technologies interactions are such that the spillover operator is unitary similar to the multiplication one, it is always possible to redefine 'technologies' in such a way, that they become separated and the R&D system (33) admits solution as an infinite-dimensional ODE system. Since in the social optimum the shadow costs of investments for every technology take into account all possible interactions, the resulting solution is welfare- maximizing and as such admits the BGP.

On the other hand, once the operator is not necessarily of the scalar type, but is compact, its spectrum contains only pointwise eigenvalues (discrete spectrum) and the continuous spectrum is restricted to zero (see e. g. Kolmogorov and Fomin (1999)). Thus even if the spillover operator contains the non-zero nilpotent part, this last has at most countable non-zero entries which can be corrected for by appropriate subsidies/taxes (see Bondarev and Krysiak (2017) for details).

Once the operator is not compact, the nilpotent part may contain continuous spectrum components and usual regulation cannot correct for cascading cross-technologies spillovers. Thus the first-best solution will eventually be unstable and the BGP as of Definition 2 would not exist. □

# C Proof of Proposition 4

*Proof.* For all $t$ when $\Theta|_{t\in\mathcal{F}} = \mathbf{S}$, the BGP exists by Proposition 2. Since there are no interactions between technologies not accounted for by individual R&D firms, the welfare theorems grant the optimality of the decentralized solution.

As soon as $\Theta|_{t\in\mathcal{F}} \neq \mathbf{S}$, there exist at least some technologies with interactions not accounted for by market participants. This leads to the non-balanced development of the mass of technologies, whereas some are more efficient then others. Resources are thus concentrated by the market in the most efficient ones, while other technologies start to degrade. Even if at the next time instance it happens that $t \in \mathcal{F}$, the distortion caused on the previous step by the interactions is not smoothed out, since the competitive advantage builds up. Then the economy will slip off the decentralized BGP and cannot return there without government intervention. Thus as soon as $t = t_S$ as defined above, and operator's projection is no longer scalar(-type), the decentralized BGP is destroyed and cannot be recovered without government intervention. □

# D Proof of Proposition 5

*Proof.* 1. Follows from Proposition 3: once $\Theta$ remains compact, the socially optimal BGP exists and is feasible. Thus without further restrictions on subsidies it can be implemented.

2. If $\Theta$ is not compact, the social BGP does not exist by Prop. 3. The spectral decomposition implies that spectrum of $\Theta$ consists exactly of three parts: pointwise, continuous and residual components. By assertion the residual component is empty, thus only pointwise and continuous components have to be considered. Now observe that (optimal) subsidy is proportional to the eigenvalues of $\Theta$: indeed, these measure the difference between social and private values of the spillover as in Bondarev and Krysiak (2017). Still continuous spectrum components cannot be precisely defined and only approximate eigenvalues may exist for this part of the spectrum (and does not necessarily exist). Thus any government subsidy policy will differ from

the potential optimal to the extent of the continuous spectrum. The sustained growth path which is approximated is exactly the one with all the spillovers being internalized and the proximity to it is then defined by the size of the continuous component of the spectrum.

3. If the residual component of the spectrum is not empty, operator becomes not dense in some part of its domain. This means there are sparse spillovers where the source cannot be even approximated. Thus the optimal SGP cannot be approximated either. Still by correcting for the overall effect of cross-technologies spillovers, the government may achieve some SGP (not close to the optimal one) which is an improvement over not following SGP at all (since positive welfare at infinite horizon is always better then the finite-time collapse of the economy).

□

# E Proof of Proposition 6

*Proof.* 1. Under this assertion the operator can be made similar to the multiplication one (diagonal). Thus all the spillovers may be delineated through appropriate choice of the basis (eigenbasis). At the same time all spillovers are inessential in a sense that they do not grant crucial competitive advantage to the associated technology. Thus the symmetry is fully restores by the proper rearrangement of property rights (basis change).

2. Under this assertion the operator can be made diagonal but some technologies experience high positive spillovers which make then more efficient than others. For each such a technology $i_1$ the associated eigenvalue measures the intensity of its impact and in the renormalized form of the spillover $\Theta$ this exactly corresponds to its competitive advantage. Duration of a subsidy is then given by the time technology stays operational.

3. If $\Theta$ contains non-zero nilpotent part, not all impacts may be reassigned through property rights (the full diagonalization is not feasible). Still if the complexity is

finite, it corresponds to finite cascades of technologies subject to the spillover and thus given the ongoing structural change, the duration of the subsidy is still finite and given by the timeframe of the cascade of technologies subjected by the impact.

4. At last if simultaneously nilpotent part is nonempty, and its complexity is infinite, there are cascades of spillovers, which are not limited to certain range of technologies, but are persistent. In this case the regulation has to be permanent.

$\square$